

# QUANTUM SPACE-TIME: DEFORMED SYMMETRIES VERSUS BROKEN SYMMETRIES<sup>1</sup>

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## ABSTRACT

Several recent studies have concerned the faith of classical symmetries in quantum space-time. In particular, it appears likely that quantum (discretized, noncommutative,...) versions of Minkowski space-time would not enjoy the classical Lorentz symmetries. I compare two interesting cases: the case in which the classical symmetries are “broken”, *i.e.* at the quantum level some classical symmetries are lost, and the case in which the classical symmetries are “deformed”, *i.e.* the quantum space-time has as many symmetries as its classical counterpart but the nature of these symmetries is affected by the space-time quantization procedure. While some general features, such as the emergence of deformed dispersion relations, characterize both the symmetry-breaking case and the symmetry-deformation case, the two scenarios are also characterized by sharp differences, even concerning the nature of the new effects predicted. I illustrate this point within an illustrative calculation concerning the role of space-time symmetries in the evaluation of particle-decay amplitudes. The results of the analysis here reported also show that the indications obtained by certain dimensional arguments, such as the ones recently considered in hep-ph/0106309 may fail to uncover some key features of quantum space-time symmetries.

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# 1 Introduction

In recent years the problem of establishing what happens to the symmetries of classical spacetime when the spacetime is quantized has taken central stage in quantum-gravity research. In particular, the symmetries of classical flat (Minkowski) spacetime are well verified experimentally, so it appears that any deviation from these symmetries that might emerge from quantum-gravity theories would be subject to severe experimental constraints. As a result symmetry tests are a key component of the programme of “Quantum-Gravity Phenomenology” [1, 2, 3].

In this lecture I focus on the faith of Lorentz invariance at the quantum-spacetime level. A large research effort has been devoted to this subject. Most of these studies focus on the possibility that Lorentz symmetry might be “broken” at the quantum level; however, I have recently shown that Lorentz invariance might be affected by spacetime quantization in a softer manner: there might be no net loss of symmetries but the structure of the Lorentz transformations might be affected by the quantization procedure [4, 5]. My primary objective here will be the one of drawing a clear distinction between the broken-symmetry and the deformed-symmetry scenarios.

## 2 Quantum-Gravity Phenomenology

Quantum-Gravity Phenomenology [1] is an intentionally vague name for a new approach to research on the possible non-classical (quantum) properties of spacetime. This approach does not adopt any particular belief concerning the structure of spacetime at short distances (*e.g.*, “string theory”, “loop quantum gravity” and “noncommutative geometry” are seen as equally deserving mathematical-physics programmes). It is rather the proposal that quantum-gravity research should proceed just in the familiar old-fashioned way: through small incremental steps starting from what we know and combining mathematical-physics studies with experimental studies to reach deeper and deeper layers of understanding of the short-distance structure of spacetime. Somehow research on quantum gravity has wondered off this traditional strategy: the most popular quantum-gravity approaches, such as string theory and loop quantum gravity, could be described as “top-to-bottom approaches” since they start off with some key assumption about the structure of spacetime at the Planck scale and then they try (with limited, vanishingly small, success) to work their way back to the realm of doable experiments. With Quantum-Gravity Phenomenology I would like to refer to all studies that are somehow related with a “bottom-to-top approach” to the quantum-gravity problem.

Since the problem at hand is really difficult (arguably the most challenging problem ever faced by the physics community) it appears likely that the two complementary approaches might combine in a useful way: for the “bottom-to-top approach” it is important to get some guidance from the (however tentative) indications emerging from the “top-to-bottom approaches”, while for “top-to-bottom approaches” it might be very useful to be alerted by quantum-gravity phenomenologists with respect to the type of new effects that could be most stringently tested experimentally (it is hard for “top-to-bottom approaches” to obtain a complete description of low-energy physics, but perhaps it would be possible to dig out predictions on some specific spacetime features that appear to have special motivation in light of the corresponding experimental sensitivities).

Until very recently the idea of a Quantum-Gravity Phenomenology, and in particular of attempts of identification of experiments with promising sensitivity, was very far from the main interests of quantum-gravity research. One isolated idea had been circulating from the mid 1980s: it had been realized [6, 7, 8] that the sensitivity of CPT tests using the neutral-kaon system has improved to the point that even small effects of CPT violation originating at

the Planck scale<sup>2</sup> might in principle be revealed. These pioneering works on CPT tests were for more than a decade the only narrow context in which the implications of quantum gravity were being discussed in relation with experiments, but over the last 3 years several new ideas for tests of Planck-scale physics have appeared at increasingly fast pace, leading me to argue [1] that the times might be right for a more serious overall effort in Quantum-Gravity Phenomenology. At the present time there are several examples of experimentally accessible contexts in which conjectured quantum-gravity effects are being considered, including studies of in-vacuo dispersion using gamma-ray astrophysics [9, 10], studies of laser-interferometric limits on quantum-gravity induced distance fluctuations [11, 12], studies of the role of the Planck length in the determination of the energy-momentum-conservation threshold conditions for certain particle-physics processes [13, 14, 15, 16, 17], and studies of the role of the Planck length in the determination of particle-decay amplitudes [18]. These experiments might represent the cornerstones of quantum-gravity phenomenology since they are as close as one can get to direct tests of space-time properties, such as space-time symmetries. Other experimental proposals that should be seen as part of the quantum-gravity-phenomenology programme rely on the mediation of some dynamical theory in quantum space-time; comments on these other proposals can be found in Refs. [1, 19, 20, 21, 22, 23].

The primary challenge of quantum-gravity phenomenology is the one of establishing the properties of space-time at Planckian distance scales. However, there is also recent discussion of the possibility that quantum-spacetime effects might be stronger than usually expected, *i.e.* with a characteristic energy scale that is much smaller (perhaps just in the TeV range!) than the Planck energy. Examples of mechanisms leading to this possibility are found in string-theory models with large extra dimensions [24] and in certain noncommutative-geometry models [25]. The study of the phenomenology of these models of course is in the spirit of quantum-gravity phenomenology, although it is of course less challenging than the quantum-gravity-phenomenology efforts that pertain effects genuinely at the Planck scale.

### 3 The faith of Lorentz symmetry in quantum space-time

If the Planck length,  $L_p$ , only has the role we presently attribute to it, which is basically the role of a coupling constant (an appropriately rescaled version of the coupling  $G$ ), no problem arises for FitzGerald-Lorentz contraction, but if we try to promote  $L_p$  to the status of an intrinsic characteristic of space-time structure (or a characteristic of the kinematic rules that govern particle propagation in space-time) it is natural to find conflicts with FitzGerald-Lorentz contraction.

For example, it is very hard (perhaps even impossible) to construct discretized versions or non-commutative versions of Minkowski space-time which enjoy ordinary Lorentz symmetry. Pedagogical illustrative examples of this observation have been discussed, *e.g.*, in Ref. [26] for the case of discretization and in Refs. [27, 28] for the case of non-commutativity. Discretization length scales and/or non-commutativity length scales naturally end up acquiring different values for different inertial observers, just as one would expect in light of the mechanism of FitzGerald-Lorentz contraction.

There are also dynamical mechanisms (of the spontaneous symmetry-breaking type) that can lead to deviations from ordinary Lorentz invariance, it appears for example that this might be possible in string field theory [29].

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<sup>2</sup>The possibility of Planck-scale-induced violations of the CPT symmetry has been extensively considered in the literature. One simple point in support of this possibility comes from the fact that the CPT theorem, which holds in our present conventional theories, relies on exact locality, whereas in quantum gravity it appears plausible to assume lack of locality at Planckian scales.

Departures from ordinary Lorentz invariance are therefore rather plausible at the quantum-gravity level. Here I want to emphasize that there are at least two possibilities: (i) Lorentz invariance is broken and (ii) Lorentz invariance is deformed.

### 3.1 Deformed Lorentz invariance

In order to be specific about the differences between deformed and broken Lorentz invariance let me focus on the dispersion relation  $E(p)$  which will naturally be modified in either case. Let me also assume, for the moment, that the deformation be Planck-length induced:  $E^2 = m^2 + p^2 + f(p, m; L_p)$ . If the function  $f$  is nonvanishing and nontrivial and the energy-momentum transformation rules are ordinary (the ordinary Lorentz transformations) then clearly  $f$  cannot have the exact same structure for all inertial observers. In this case one would speak of an instance in which Lorentz invariance is broken. If instead  $f$  does have the exact same structure for all inertial observers, then necessarily the transformations between these observers must be deformed. In this case one would speak of an instance in which the Lorentz transformations are deformed, but Lorentz invariance is preserved (in the deformed sense).

While much work has been devoted to the case in which Lorentz invariance is actually broken, the possibility that Lorentz invariance might be deformed was introduced only very recently by this author [4, 30, 31, 5, 32]. An example in which all details of the deformed Lorentz symmetry have been worked out is the one in which one enforces as an observer-independent statement the dispersion relation

$$L_p^{-2} \left( e^{L_p E} + e^{-L_p E} - 2 \right) - \vec{p}^2 e^{-L_p E} = m^2 \quad (1)$$

In leading (low-energy) order this takes the form

$$E^2 - \vec{p}^2 + L_p E \vec{p}^2 = m^2 . \quad (2)$$

The Lorentz transformations and the energy-momentum conservation rules are accordingly modified [5].

### 3.2 Broken Lorentz invariance

The case of broken Lorentz invariance requires fewer comments since it is more familiar to the community. In preparation for the analysis reported in the next Section it is useful to emphasize that the same dispersion relation (2), which was shown in Refs. [4, 5] to be implementable as an observer-independent dispersion relation in a deformed-symmetry scenario, can also be considered [9] as a characteristic dispersion relation of a broken-symmetry scenario. In this broken symmetry scenario the dispersion relation (2) would still be valid but only for one “preferred” class of inertial observers (*e.g.* the natural CMBR frame) and it would be valid approximately in all frames not highly boosted with respect to the preferred frame. In highly-boosted frames one might find the same form of the dispersion relation but with different value of the deformation scale (different from  $L_p$ ). All this follows from the fact that in the broken-symmetry scenario the laws of transformation between inertial observers are unmodified. Accordingly also energy-momentum conservation rules are unmodified.

Another scenario in which one finds broken Lorentz invariance is the one of canonical noncommutative spacetime, in which the dispersion relation is modified (with different deformation term [33, 34]), but, again, the energy-momentum Lorentz transformation rules are not modified.

## 4 Illustrative example: photon-pair pion decay

In order to render very explicit the differences between the broken-symmetry and the deformed-symmetry case in this Section I consider photon-pair pion decay adopting in one case deformed energy-momentum conservation [5], as required by the deformed Lorentz transformations of the deformed-symmetry case, and in another case ordinary energy-momentum conservation, as required by the fact that the Lorentz transformation rules are unmodified in the broken-symmetry case, but for both cases I impose **the same** dispersion relation (2).

In the broken-symmetry case, combining (2) with ordinary energy-momentum conservation rules, one can establish a relation between the energy  $E_\pi$  of the incoming pion, the opening angle  $\theta$  between the outgoing photons and the energy  $E_\gamma$  of one of the photons (the energy  $E'_\gamma$  of the second photon is of course not independent; it is given by the difference between the energy of the pion and the energy of the first photon):

$$\cos(\theta) = \frac{2E_\gamma E'_\gamma - m_\pi^2 + 3L_p E_\pi E_\gamma E'_\gamma}{2E_\gamma E'_\gamma + L_p E_\pi E_\gamma E'_\gamma}, \quad (3)$$

where indeed  $E'_\gamma \equiv E_\pi - E_\gamma$ . This relation shows that at high energies (starting at energies of order  $(m_\pi^2/L_p)^{1/3}$ ) the phase space available to the decay is anomalously reduced: for given value of  $E_\pi$  certain values of  $E_\gamma$  that would normally be accessible to the decay are no longer accessible (they would require  $\cos\theta > 1$ ).

In the deformed-symmetry case one enforces the deformed conservation rules [5]

$$E_\pi = E_\gamma + E'_\gamma, \quad \vec{p}_\pi = \vec{p}_\gamma + \vec{p}_{\gamma'} + L_p E_\gamma \vec{p}_{\gamma'}, \quad (4)$$

which, when combined again with (2), give raise to the different relation

$$\cos(\theta) = \frac{2E_\gamma E'_\gamma - m_\pi^2 + 3L_p E_\gamma^2 E'_\gamma + L_p E_\gamma E_\gamma'^2}{2E_\gamma E'_\gamma + 3L_p E_\gamma^2 E'_\gamma + L_p E_\gamma E_\gamma'^2}. \quad (5)$$

Here it is easy to check that for all physically acceptable values of  $E_\gamma$  (given the value of  $E_\pi$ ) one is never led to consider the paradoxical condition  $\cos\theta > 1$ : there is no severe implication of the deformed-symmetry case for the amount of phase space available for the decays (certainly not at energies around  $(m_\pi^2/L_p)^{1/3}$ , possibly at Planckian energies).

## 5 Closing remarks

As shown by the illustrative example of calculation presented in the preceding Section, the differences between the case in which Lorentz invariance is broken and the case in which Lorentz invariance is deformed can be very significant also quantitatively, concerning the nature and the magnitude of the effects predicted, besides being quite clearly significant at the conceptual level.

The calculation in the preceding Section also shows that simple dimensional estimates of the effects induced by deviations from Lorentz invariance are futile. Both in the broken-symmetry case and in the deformed-symmetry case the Planck-scale deformation introduces the same correction terms, respectively in Eqs. (3) and (5), but in the broken-symmetry case I found profound implications for pion decay, whereas in the deformed-symmetry case the correction terms arranged themselves in a less “armful” manner. This observation appears to be particularly significant for the argument recently presented by Brustein, Eichler and

Foffa in Ref. [35]: in that paper, by applying dimensional analysis to some aspects of neutrino physics it was suggested that Planck-scale-induced deviations from ordinary Lorentz invariance are unlikely. The fact that the analysis reported in Ref. [35] relies on the type of dimensional arguments which I have here shown to be inclusive, forces us to assume that the conclusions drawn in Ref. [35] are equally unreliable. Certainly the observations reported in Ref. [35] provide strong motivations for future dedicated and rigorously quantitative studies of the relevant aspects of neutrino physics within specific examples of Planck-scale-induced deviations from ordinary Lorentz invariance.

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